

## **APPLICATION OF FIXATOR-NORATOR PAIR IN ANALOG CIRCUIT DESIGN FOR BANDWIDTH**

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### **Abstract**

The article presents a method for the design of frequency response of analog circuits employing companion modelling and Fixator-Norator Pairs (FNP). The proposed method can cover a wide range of cases but this study limits itself to redesigning bandwidth of amplifiers. FNP finds application in a wide area of analog design but the pair faces difficulty when reactive components are included in the circuit. The proposed method overcomes such a limitation by using companion modelling to generate a linear equivalent circuit, in which FNP can be directly applied. The assumption is that a reactive component must be designed using FNP and inserted in to a proper location in the target circuit so that its frequency response becomes close to the desired one. Finally, three examples are worked out for demonstrating the methodology.

Keywords: Analog circuits, Fixator-norator pair, Companion modelling, Storage elements, Bandwidth.

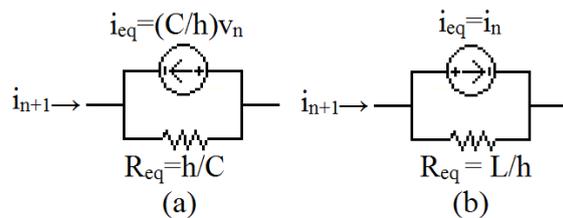
## 1. Introduction

Designs of analog circuits involve different aspects; among them, the design of reactive components needs much care. Resistors possess almost similar linear behaviour to both AC and DC signals. Reactive components like capacitors and inductors, on the other side, have their own frequency response and behave differently at different frequencies. This makes the design process difficult and complex. Design, analysis and fault diagnosis of analog circuits are widely studied in literature [1, 2]. The new advances in symbolic analysis can ease the analog design steps and further studies may make them adaptable to frequency response designs in future [3-5].

In the recent years, nullor elements show wide range of applications, not limited to modelling active elements [4, 6], and in low sensitivity current mode filters [7]. Nullors and pathological mirror elements show wide applications in many circuit designs [8-15]. Synthesis of analog circuits and modelling of active devices are some of their popular applications [15-17]. Nullors are also applicable for parametric fault detection [18]. Active networks containing nullors can be solved with two-graph method [19]. A combination of nullor and sources - the Fixator-Norator Pair (FNP) [20-24] is now getting attraction due to its simplicity and ease of usability in analog designs. FNPs are found useful in source allocation, biasing design [23] and in the design of active loads and current mirrors. But in most cases, the pair assumes circuits free of reactive components for its operation. This article presents a method to design circuits having reactive components by making use of FNP and companion modelling [25].

## 2. Companion Models

The storage elements capacitors and inductors can be modelled using Forward Euler, Backward Euler (B.E) and Trapezoidal methods [25]. Here we are using BE model to demonstrate the methodology. A capacitor or an inductor at a particular time instant  $t_{n+1}$  can be represented by a resistor and a parallel current source as shown in Fig. 1. The value of resistance and current source are depend upon the time step  $h$ , the former is kept fixed for all value of  $t$  and later is updated at every  $t$  for fixed  $h$ . Such a circuit now becomes a linear one, so that FNPs can be directly applied to the resultant circuit.



**Fig. 1. Companion models (a) Capacitor, (b) Inductor.**

The BE equation for a capacitor is,

$$v_{n+1} = v_n + \frac{h}{C}(i_{n+1}) \quad (1)$$

Thus the storage elements in a circuit at a particular time instant  $t_{n+1}$  can be replaced with circuits in Fig. 1, where,  $i_{n+1}$  and  $v_{n+1}$  are the current through and voltage across the storage element at time  $t_{n+1}$ . Similarly  $i_n$  and  $v_n$  represent the current through and voltage across the storage element at time  $t_n$ . Also,  $C$  and  $L$  represent the capacitance and inductance of the storage elements respectively and  $h$  is the time step. Such a circuit is now equipped to insert FNPs for a target design. The application of this method is wide but here we are studying only the design of bandwidth of an amplifier to understand the methodology.

### 3. Design Procedure for a Given Frequency Response

Here, our objective is to modify a given analog circuit so as to obtain a desirable frequency response. Modifications in the sense add/update a reactive component such as capacitor or inductor in the given circuit. The generic design procedure is given in the following algorithm for the design of capacitor. A similar procedure can follow for the design of inductors.

*Algorithm:*

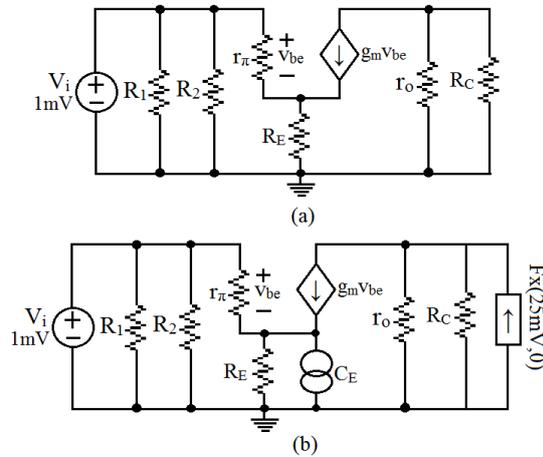
- i. Create the linear equivalent model of the circuit under consideration with all the active elements replaced with their small signal models.
- ii. Select a particular point in the model response; the corresponding frequency and magnitude can be used to calculate the circuit's output at this point in volt.
- iii. For the selected frequency, split its time period into smaller time steps each of width  $h$ . It is now easy to find the magnitude of input for the required output for any  $t$  at the selected frequency.
- iv. Apply a DC voltage equal to the input voltage calculated in step 3 at a selected  $t$  to the input of linear circuit. Apply fixator of value equal to required output at this  $t$  to the output of the linear circuit; its pairing norator does the role of the feedback capacitor.
- v. Simulate the circuit. Norator voltage and norator current represent to  $v_{n+1}$  and  $i_{n+1}$  respectively of Eq. (1). Now, value of  $C$  can be easily found.

Before going further, note that the decision on selection and location of reactive components depends on the skill of the designer. Also, we have a circuit with undesirable frequency response; the desired response must be achievable by adding (or updating) reactive components to it. Following examples demonstrate the methodology.

### 4. Case study

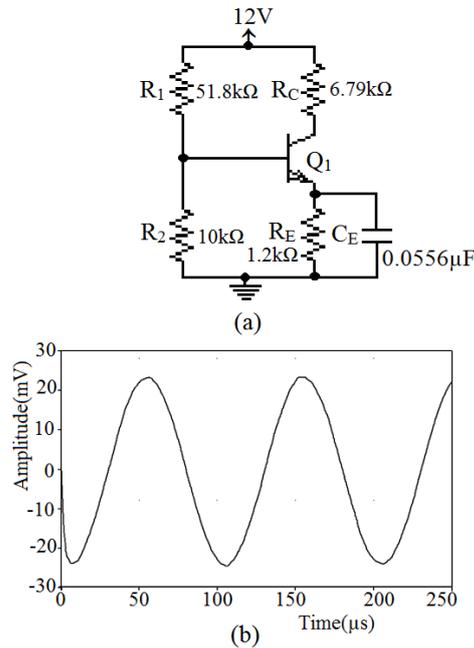
Example 1: In this section, we are implementing FNPs to upgrade the gain of a common emitter (CE) amplifier. An emitter bypass capacitor provides an AC ground. This capacitor prevents the AC signal from being dropped across the emitter resistor, and the net effect is a higher gain of magnitude  $-g_m R_C$ . In addition, such a capacitor also provides AC ground to the high frequency noise present in the circuit. Therefore, an increase in gain can be made possible by adding suitable emitter bypass capacitor. Figure 2(a) shows the small signal model of a common emitter amplifier at time  $t=75\mu s$ . Without having a bypass capacitor, it has a gain of 5.75 for an input signal of frequency 10 KHz. Our aim is to increase the gain to about 25 by using a bypass capacitor. Figure 2(b) shows the design process by using

FNP. Here,  $R_1 = 51.8 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $R_C = 6.79 \text{ k}\Omega$  and  $R_E = 1.2 \text{ k}\Omega$ . The transistor parameters  $r_o$  is taken as  $70 \text{ k}\Omega$  and  $r_\pi$  is  $2.194 \text{ k}\Omega$ . Simulating Fig. 2(b) gives value of  $i_{n+1} = 3.325 \text{ }\mu\text{A}$  and  $v_{n+1} = 882 \text{ }\mu\text{V}$ .



**Fig. 2. (a) Small signal model of a  $C_E$  amplifier without a bypass capacitor, (b) Designing of  $C_E$  for a desired gain.**

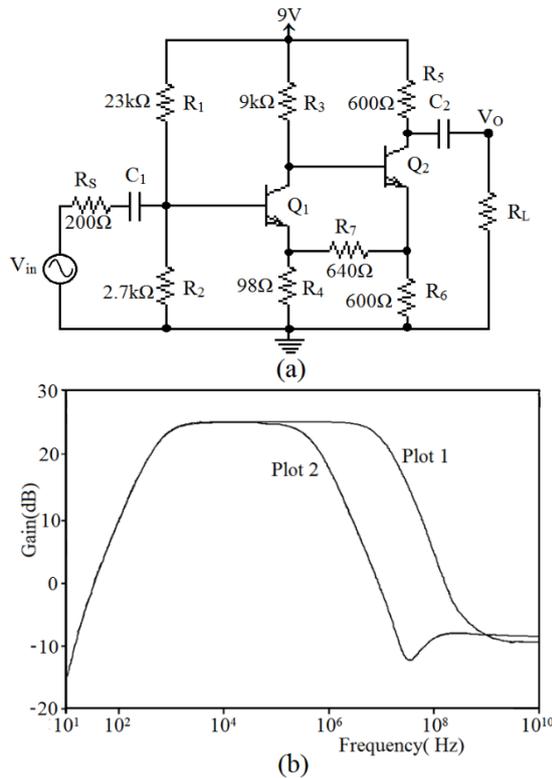
Using Eq. (1), we can calculate the value of  $C_E$  as  $0.0556 \text{ }\mu\text{F}$ , with  $h = 1 \text{ }\mu\text{s}$  and  $v_n = 823.3 \text{ }\mu\text{V}$ . It is now necessary to verify our design result by simulating the common emitter amplifier. Figure 3(a) shows the final circuit and Fig. 3(b) shows the result obtained by simulating the circuit with an emitter bypass capacitor of  $0.0556 \text{ }\mu\text{F}$  and an input of  $\sin(20000\pi t) \text{ mV}$ .



**Fig. 3. Common emitter amplifier: (a) modified circuit, (b) its response.**

Note: The emitter bypass capacitor, as the name suggests, bypasses the AC signal from being dropped across the emitter resistance. The final result is an increase in gain. Here, fixator  $F_x$  (25 mV, 0) keep the output port at 25 mV and at the same time its pairing norator act as a placeholder for the bypass capacitor. That is for an input of 1 mV, the output is kept at 25 mV and norator will conduct the required current to maintain the output port at the desired level.

Example 2: Given a two-stage BJT amplifier as shown in Fig. 4(a). The objective in this example is to modify the frequency response and bandwidth of the amplifier. The response of amplifier is shown in Plot 1, Fig. 4(b), and it is clear that the bandwidth of this amplifier is very high. Plot 2 shows the required frequency response expected from the modified circuit. The responses in Plot 1 and 2 are almost same except for the change in higher cut off frequency.



**Fig. 4. Two-stage BJT amplifier: (a) circuit diagram, (b) its original and expected responses.**

The response as in Plot 2 can be obtained by adding a feedback capacitor from output back to input, that is between collector of  $Q_2$  and base of  $Q_1$ . In this case we have selected frequency  $f=1$  kHz. For an input of 50 mV, as per Plot 2, its peak output is calculated as 600 mV at  $t=0.25$  ms. We can use the circuit in Fig. 5 to find the value of  $C_F$ . Simulating the circuit gives values for  $v_{n+1}$  and  $i_{n+1}$ . Using Eq. (1),  $C_F$  can be calculated as 0.095 nF. Figure 6(a) shows the final circuit of the modified amplifier and Fig. 6(b) shows its output for an input  $50 \sin(2000\pi t)$  mV.

Note: Reactive elements have a major role in deciding the behaviour of a circuit to AC signals. Taking the case of capacitors, there are mainly of two types, series and parallel. Coupling and by pass capacitors are generally in a series path to the signal flow. They are high value type and can affect the lower cut-off frequency of amplifiers and filters. Some other low value capacitors such as internal transistor capacitance are usually in a path parallel to signal flow. Such capacitors decide the higher cut-off frequency of amplifier/filter circuits. Thus, if we want to alter the lower cut-off frequency, we need to alter the value of coupling or bypass capacitors. On the other hand, if we need to modify the higher cut-off frequency, we must consider altering transistor capacitors. In this problem, a parallel capacitor (or feed back in this case) is designed, which can reduce the higher cut-off frequency. They act as a high frequency switch, and conduct extra current to reduce the gain at the desired frequency and thus the higher cut-off frequency. A similar analysis can be done in example 3 also.

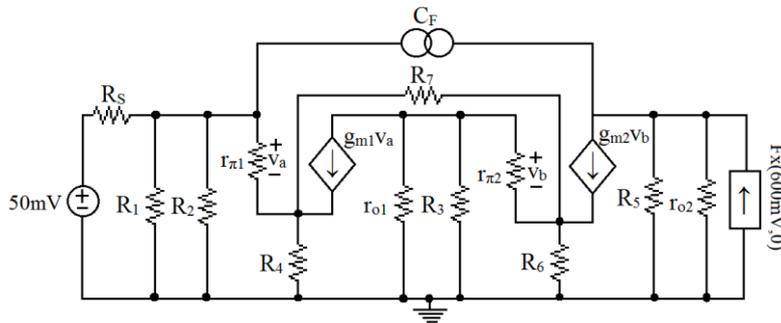


Fig. 5. Design of  $C_F$  using FNP.

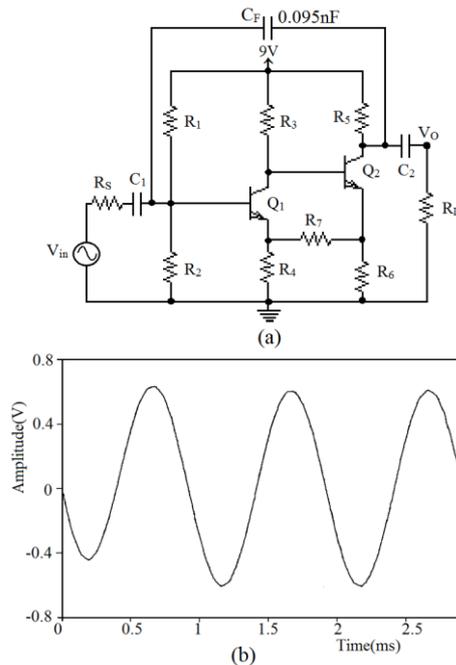


Fig. 6. BJT amplifier: (a) modified circuit, (b) its response.

Example 3: Let us consider a well-designed MOS op-amp as shown in Fig. 7(a) for redesigning. This amplifier has a bandwidth in MHz range as shown in Fig. 7(b). But for some application, we need this amplifier to filter out frequencies above 1 kHz. Now the question is how to modify the op-amp circuit for the desirable frequency response? Again the answer is FNP method. Inserting a capacitor of sufficient value parallel to the current mirror of the op-amp circuit (i.e., across  $M_5$ ) can produce the required frequency response. Following the algorithm stated in section 3, we need the linear equivalent model of the op-amp circuit. From Fig. 7(b) maximum gain of the circuit is 580 V/V. For the desired response, we need a gain of 410 V/V at 1 kHz. Therefore, give an input of 10 mV to the linear equivalent circuit and apply fixator (-4.1V, 0) to its output port. The pairing norator does the role of a parallel capacitor across the current mirror. The circuit arrangement for the design process is shown in Fig. 8.

On simulating the circuit shown in Fig. 8, the norator current and voltage are obtained as 2.536  $\mu$ A and 69.59 mV respectively. Using Eq. (1), we can calculate the value of  $C_1$  as 54 nF. Finally, it is necessary to test the correctness of our design by simulating the updated circuit. Figure 9(a) shows the updated circuit and Fig. 9(b) its response.

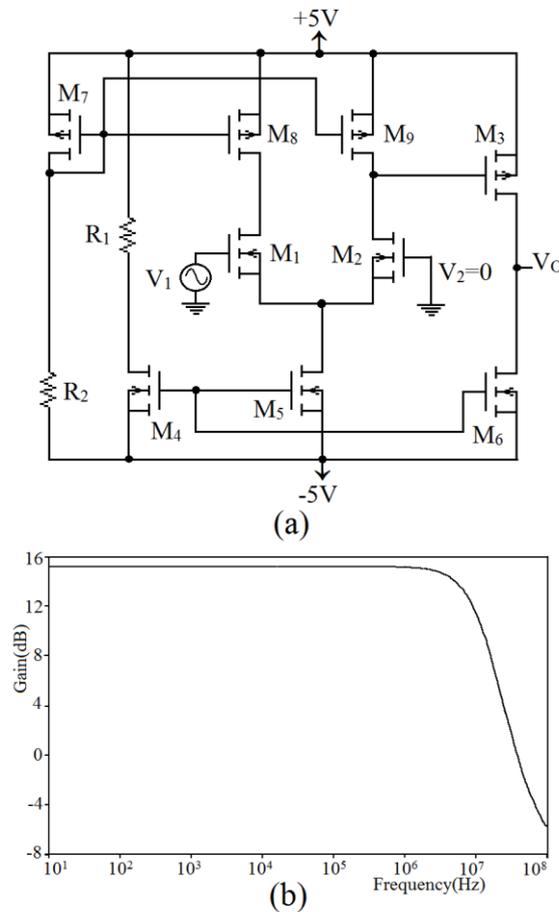


Fig.7. (a) MOS op-amp, (b) its frequency response.



the proposed technique, we are generating a linear equivalent model of the actual circuit by replacing the reactive elements with their companion models. All the other parameters in the linear circuit are maintained equal to their actual values, so a fruitful result will be obtained with the circuit. The design specs are inserted only at this stage, so that the linear circuit is useful even if the design conditions are changed. Moreover, FNP allows flexibility in design and a skilful designer can render the design problem into proper sets of FNPs, so that the design effort and time should greatly decrease.

## 6. Conclusions

Design of frequency response of analog circuits has been done using companion modelling and FNP. A linear equivalent circuit of the targeted circuit is generated and by using the proposed method, designer can modify the frequency response of analog circuit to match with a desired one. For this purpose, a reactive component is designed and applied to proper place in the target circuit. The usefulness of the proposed method is demonstrated using three examples, which are design of gain of a CE amplifier, design of bandwidth for a BJT amplifier circuit and redesign of a MOS op-amp and the results are verified. This study offers a generic approach towards circuit designs dealing with reactive components and can be extended to wide range of analog designs.

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### Nomenclatures

$AL$	Active load
$C$	Capacitance, F
$CM$	Current mirror
$gm$	Transconductance, S
$h$	Time step, s
$I_n$	Current at time $t=t_n$ , A
$L$	Inductance, H
$R$	Static resistance, $\Omega$
$r$	Dynamic resistance, $\Omega$
$r_o$	Output resistance of transistor, $\Omega$
$r_\pi$	Base-emitter resistance of transistor, $\Omega$
$t$	Time instant, s
$V_n$	Voltage at time $t=t_n$ , V

### Abbreviations

BE	Backward Euler
BJT	Bipolar Junction Transistor
CE	Common Emitter
FNP	Fixator Norator Pair
MOS	Metal Oxide Semiconductor

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